The fully self-consistent charge-exchange quasiparticle random phase approximation and its application to the isobaric analog resonances

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Abstract

A microscopic model aimed at the description of charge-exchange nuclear excitations along isotopic chains which include open-shell systems, is developed. It consists of quasiparticle random phase approximation (QRPA) made on top of Hartree-Fock-Bardeen-Cooper-Schrieffer (HF-BCS). The calculations are performed by using the Skyrme interaction in the particle-hole channel and a zero-range, density-dependent pairing force in the particle-particle channel. At variance with the (many) versions of QRPA which are available in literature, in our work special emphasis is put on the full self-consistency. Its importance, as well as the role played by the charge-breaking terms of the nuclear Hamiltonian, like the Coulomb interaction, the charge symmetry and charge independence breaking (CSB-CIB) forces and the electromagnetic spin-orbit, are elucidated by means of numerical calculations of the isobaric analog resonances (IAR). The theoretical energies of these states along the chain of the Sn isotopes agree well with the experimental data in the stable isotopes. Predictions for unstable systems are presented.

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I. INTRODUCTION

The significant lack of knowledge concerning many properties of the charge-exchange nuclear excitations contrasts markedly with their importance for nuclear structure and the impact which they have on many interesting physical phenomena.

The charge-exchange transitions involve a change in N and Z of the nucleus, keeping A fixed. They can take place spontaneously, like in the well-known case of β-decay, or be induced by external fields when in a nuclear reaction a given amount of excitation energy ΔE and angular momentum ΔJ is released to the nucleus. The spectra of charge-exchange reactions, like (p,n) or $(^3\mathrm{He},t)$, are characterized by the emergence of collective isovector (i.e., ΔT=1) giant resonances (IVGRs) in analogy with the non charge-exchange case [1]. However, a unifying picture of these ΔT_z=$\pm$1 states is still, to a large extent, missing. For instance, the ΔL=0 charge-exchange isovector giant monopole resonance (IVGMR) is one of the most elusive nuclear states, despite a long series of experiments aimed at its identification [2]; at the same time, its knowledge would be important for the determination of the ground state isospin mixing. Also the higher multipoles, that is, the charge-exchange dipole, quadrupole and octupole resonances, are basically unknown. This is mainly due to the lack of really selective probes: in particular, the separation of the electric (i.e., ΔS=0 or "non spin-flip") and magnetic (i.e., ΔS=1 or "spin-flip") modes is far from being trivial. On the other hand, a systematic pattern of the energy and collectivity of these states would shed light on the strong uncertainties concerning the isovector part of the nucleon-nucleon (NN) effective interaction and the symmetry part of the nuclear equation of state.

It has to be mentioned that knowing the properties of the nuclear charge-exchange states allows also to attack other kinds of problems outside the realm of nuclear structure. These states enter the description of the double β-decay, and the need of a reliable theory of this process is a longstanding problem. More generally, all the weak interaction processes within atomic nuclei involve charge-exchange transitions as far as charged currents are involved. We have in mind many processes which are of interest for neutrino physics, like the interaction of these peculiar particles with nuclei, or for astrophysics, like the mechanisms which are responsible for the evolution of neutron stars or the β-decay of isotopes which lie on the r-process path of stellar nucleosynthesis.

A significant exception to the unsatisfactory ignorance of the charge-exchange IVGRs is

provided by the availability of many experimental data on the isobaric analog resonance (IAR) and the Gamow-Teller resonance (GTR). The IAR is the simplest charge-exchange transition, in which a neutron is changed into a proton without any other variation of the quantum numbers (that is, $\Delta J{=}\Delta L{=}\Delta S{=}0$). The corresponding operator is

$$\hat{O}_{\rm IAR} \equiv \sum_{i=1}^{A} t_-(i), \tag{1}$$

namely it is the usual Fermi, or isospin-lowering, operator. In the Gamow-Teller case, the transition is accompanied by a spin-flip ($\Delta L{=}0$, $\Delta J{=}\Delta S{=}1$), and the operator is

$$\hat{O}_{\rm GTR} \equiv \sum_{i=1}^{A} \vec{\sigma} t_-(i). \tag{2}$$

Many data coming at an early stage from (p,n), and later from other reactions, have shown that these resonances can be systematically identified in the isotopes with neutron excess (in which the corresponding t_+ transitions are Pauli-blocked). The IAR consists of a single, very narrow peak, whereas the GTR manifests itself with a broad bump and can also be fragmented in different peaks. Experimentally, when the incident projectile energy is increased, the excitation of the GTR is favoured over the IAR; this experimental fact has allowed to establish that the strengths of the spin-independent and the spin-dependent components of the effective NN interaction have different behaviour as a function of the energy.

From this rather general introduction, the motivation for microscopic calculations of the charge-exchange states in nuclei is already evident. We must add that one of the main present interests in nuclear physics is the understanding of the limits of nuclear stability, and of the exotic, very neutron-rich (or proton-rich) nuclei, that is, of the systems with different values of $N - Z$ than those which characterize the valley of stability. The experimental evidences about the isospin properties of exotic nuclei are still rather scarce. In order to make predictions in this delicate sector accurate calculations are called for, which do not make approximations by neglecting terms of the nuclear Hamiltonian in an uncontrolled way.

For nuclei with mass up to A $\sim$50, the shell model (SM) calculations can be rather successful and are indeed performed, also in the cases of interest for applications. The agreement with the experimental findings (like the GT strength and/or the β-decay half-life) can be quite good [3]. However, these calculations become too demanding, or impossible, for heavier nuclei. Also, they have trouble if the space must be large enough so to account for

high-energy transitions; these transitions can be induced, for instance, by neutrinos which follow a supernova explosion. In Ref. [4] it has been shown that for energies above 50 MeV the SM calculations may underestimate the strength of the charge-exchange transitions.

The alternative choice is a mean-field based calculation which employs an effective NN interaction. In this case, the ground state of the parent (N,Z) system is obtained by means of a Hartree-Fock (HF) calculation, extended to Hartree-Fock-Bardeen-Cooper-Schrieffer (HF-BCS) or Hartree-Fock-Bogoliubov (HFB) in the case of open-shell nuclei where pairing is relevant. In the two cases, respectively, the charge-exchange excited states in the $(N \mp 1, Z \pm 1)$ isobars can be obtained within the framework of the linear response theory, that is, by using the random phase approximation (RPA) or its extension to the pairing case, namely the quasiparticle RPA (QRPA). These are well-known theories, whose general features can be found in many textbooks. However, there are only few examples, if any, of fully self-consistent QRPA calculations — which constitute the proper scheme for the analysis of long isotopic chains extending towards the drip lines. In fact, self-consistency is a crucial issue if the calculations are required to have predictive power far from the experimentally known regions of the mass table. Moreover, as we discuss below, self-consistency plays a special role if the isospin symmetry and its breaking enters the discussion. We repeat here that self-consistency means that the residual particle-hole (p-h) and particle-particle (p-p) residual forces, which enter the QRPA equations (cf. Sec. II), are derived from the same energy functional from which the HF-BCS of HFB equations describing the ground state are obtained.

The first attempt of self-consistent QRPA on top of HFB is found in Ref. [5]. The Skyrme zero-range force and a zero-range pairing interaction are used, respectively, in the mean-field and in the pairing channel to solve the HFB equations in coordinate space (cf. also Ref. [6]). The associated QRPA equations are solved in the canonical basis. The method is applied to the calculation of Gamow-Teller β-decay half-lives. These 1^+ states are known to be sensitive only to the T=0 component of the residual p-p interaction, if pairing is described by means of a zero-range force. In Ref. [5] it is assumed that, since this T=0 pairing does not manifest itself in the HFB ground state of nuclei with N different from Z by a few units, one is free to introduce it within QRPA in a completely different way than the T=1 pairing, without any constraint related to self-consistency. The authors have employed a finite-range interaction with free parameters: the overall strength is fitted to reproduce some

selected β-decay experimental findings. The same approach is used in Ref. [7] to analyze the performance of existing Skyrme parametrizations in the case of the GT resonances, and to correlate it with their ability to reproduce the values of the empirical Landau parameters of infinite matter.

In the present paper, we would like to discuss the implementation of a fully self-consistent charge-exchange QRPA by putting emphasis on aspects which were not considered in Refs. [5, 7]. A first aspect is the issue of isospin invariance. We show that the T=1 component of the residual p-p force can be fixed by exploiting this invariance. Our hypothesis is supported by the absence of strong evidences coming from literature which point to a clear need to differentiate the strengths of the three components of the T=1 pairing. Within this assumption, we show that we can obtain results for the IAR which are quite satisfactory when compared with experiment. The IAR is a serious benchmark for every theoretical model, because of its intimate relationship with the isospin symmetry (cf., e.g., Ref. [8]). In fact, if the whole Hamiltonian H commuted with isospin, and if one were able to solve H exactly, the resulting IAR would be degenerate with the parent ground state. Many of the approximation schemes which are commonly used to solve the nuclear many-body problem destroy this property of the Hamiltonian. HF and HF-BCS belong to this category and introduce a spurious isospin breaking (as soon as $N \neq Z$ in case of HF). Instead, it has been demonstrated that self-consistent RPA and QRPA calculations restore the isospin symmetry and eliminate any spuriousity [9], being in this sense "good" symmetry-preserving approximations. Consequently, only within their framework it is possible to assess the relative importance of the physical contributions which are responsible for an explicit isospin breaking: the Coulomb force, the electromagnetic spin-orbit, and the other charge-symmetry breaking (CSB) and charge-independence breaking (CIB) terms in the nuclear Hamiltonian. The study of these issues in the case of the IAR for the open-shell isotopes is an original feature of the present work. Since we do not go beyond QRPA, we cannot discuss the (narrow) width of the IAR. The extensions of RPA and of the normal, non charge-exchange QRPA, intended to take into account the coupling with more complex configuration and therefore to describe the spreading width of the resonances, are described elsewhere (see the references quoted in Sec. II).

In our work, we employ zero-range forces. We are not aware of self-consistent calculations of charge-exchange states done by using finite-range interactions like Gogny. On the other

hand, in recent papers the relativistic mean-field (RMF) effective Lagrangians, based on the description of nucleons as Dirac particles which interact by means of the exchange of effective mesons, have been used for the calculation of the IAR and the GTR [10], as well as of β-decay rates [11]. The RMF description of the ground state and of the excited nuclear states emerges from rather different ingredients than those which characterize the non-relativistic mean-field. It is known that the isovector channel of the NN interaction, and consequently the symmetry part of the energy functional, are quantitavely not the same, generally speaking, in the two cases (the symmetry energy at saturation and its derivatives are generally larger in the relativistic case). In the relativistic calculations of the spin and isospin excitations the pion-exchange is very important; but this degree of freedom is not present in the ground state description because of parity conservation. On the other hand, in the case of RMF the spin-orbit is automatically considered, at variance with the non-relativistic case. Finally, we are not aware of attempts to include CSB and CIB forces in the RMF calculations. All this should be kept in mind when comparing our results with those of [10].

II. THEORETICAL FRAMEWORK

As mentioned in the previous Section, charge-exchange RPA and QRPA are well-known and described in textbooks. We try here to recall only the basic elements, or some details which are useful for the following discussion.

In the case of charge-exchange RPA, self-consistent calculations have been available for many years. In particular, the first application to the case of the IAR can be found in Ref. [12]. Extensive calculations of the response to different multipole operators, made by using the coordinate space formulation of RPA with proper treatment of the particle continuum, but dropping for simplicity some terms of the residual interaction, are reported in [13]. As we have recalled in the Introduction, it is well-known that mean-field calculations of this kind cannot reproduce the total width of the resonances, but only the escape width if the continuum is correctly taken into account. The spreading width, associated with the coupling of the simple p-h configurations to the more complex states, of two particle-two hole ($2p$-$2h$) character, can be described only by diagonalizing the effective Hamiltonian in a larger model space than the one of RPA. A microscopic model suited for this purpose has

been developed in [14, 15]. In [16] the importance of CSB and CIB forces for the IAR width has been studied.

In the case of the QRPA, most of the charge-exchange calculations performed so far make use of two separable *p-h* and *p-p* residual interactions (having, as a rule, the same functional form and two different overall parameters g_{ph} and g_{pp}), as in the pioneering work by J. A. Halbleib and R. A. Sorensen [17], where the formalism has been developed for the first time.

We start by solving the HF-BCS equations in coordinate space by using a radial mesh extending up to 20 fm (with a step of 0.1 fm). The HF equations contain the Skyrme *NN* interaction and we have chosen in this work the parametrization SLy4 [18], which has been determined by trying to retain many of the advantageous features of the previous versions of the Skyrme force, as well as by fitting the equation of state of pure neutron matter obtained by means of realistic forces. This latter characteristic should justify its use for systems outside the valley of stability. The BCS equations are solved, as usual, in a limited space: only the levels which correspond to the 82–126 neutron shell are included. The pairing force that we have used is of the type

$$V = V_0 \left(1 - \left(\frac{\varrho\left(\frac{\vec{r}_1+\vec{r}_2}{2}\right)}{\varrho_c} \right)^{\gamma} \right) \cdot \delta(\vec{r}_1 - \vec{r}_2). \tag{3}$$

The parameter γ is fixed to one for the sake of simplicity. With the same spirit, ϱ_c is set at 0.16 fm^{-3}. The strength V_0 has been determined by requiring a reasonably good agreement between the theoretical and empirical values of the pairing gaps Δ along the whole series of isotopes under study. This agreement, when V_0 is equal to our adopted value of 680 MeV fm^3, is shown in Fig. 1. We notice in this context that a rather similar pairing force, having V_0=625 MeV fm^3, has been used independently by other groups to carry out large-scale, systematic calculations of the pairing gaps and of the rotational bands (see [19] and references therein). It is known that the HFB treatment is more coherent than the HF-BCS one; however, qualitatively important differences between the results of the two methods show up only in the case of weakly bound nuclei, which will not be considered in the present study.

When the ground state is obtained, together with the filled or partially occupied states lying within the pairing window, a number of unoccupied states (which have occupation factors v^2 strictly equal to zero) are calculated by using spherical box boundary conditions. This means that our continuum is discretized. For every value of (l, j), we calculate unoc-

cupied states with six increasing values of n. The dimension of the space has been checked by looking at the results for the energy and the strength of the IAR, which have been found to be stable when we enlarge the space, by considering in some cases up to ten increasing values of n. We have checked that also the $N-Z$ sum rule is accurately reproduced. In this configuration space, the QRPA matrix equation written on the basis made up with the two quasiparticle states having good angular momentum and parity J^π, reads

$$\begin{pmatrix} A & B \\ -B & -A \end{pmatrix} \begin{pmatrix} X^{(n)} \\ Y^{(n)} \end{pmatrix} = E_n \begin{pmatrix} X^{(n)} \\ Y^{(n)} \end{pmatrix}. \tag{4}$$

In this formula, E_n is the energy of the n-th QRPA state in the parent nucleus, while $X^{(n)}$, $Y^{(n)}$ are the corresponding forward- and backward-amplitudes. The matrices A and B, in the angular momentum coupled representation, have the explicit form

$$\begin{aligned} A_{pn,p'n'} &= (E_p + E_n)\delta_{pp'}\delta_{nn'} + \\ &\quad + V^{(J)}_{pnp'n'}(u_p u_n u_{p'} u_{n'} + v_p v_n v_{p'} v_{n'}) + \\ &\quad + W^{(J)}_{pnp'n'}(u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}), \\ B_{pn,p'n'} &= -V^{(J)}_{pnp'n'}(u_p u_n v_{p'} v_{n'} + v_p v_n u_{p'} u_{n'}) + \\ &\quad + W^{(J)}_{pnp'n'}(u_p v_n u_{p'} v_{n'} + v_p u_n v_{p'} u_{n'}). \end{aligned} \tag{5}$$

Here, the indices p and p' (n and n') refer to proton (neutron) quasiparticles. E is their energy, whereas u and v are the usual BCS occupation factors. $V^{(J)}$ and $W^{(J)}$ indicate respectively the coupled p-p and p-h matrix elements. The p-h matrix elements are derived from the Skyrme part of the energy functional: all the terms are considered, including the two-body spin-orbit.

The p-p matrix elements, when consistently derived from the energy functional, are those of the bare force (3): in fact, no rearrangement terms show up if we do not impose any dependence on the anomalous density in the force itself. The isospin invariance that we have assumed, demands that the T=1 component of the pairing force is the same in the three channels: neutron-neutron, proton-neutron and proton-proton. In the present case, since we deal with the Sn isotopes which have magic proton number, there is no proton pairing in the ground state. Also, we have neglected proton-neutron pairing in the ground state: in fact, this may be important only in nuclei having $N \sim Z$ and we have considered Sn isotopes in the range $104 \leq A \leq 132$. However, the proton-neutron T=1 pairing force enters

the QRPA equations (in the V matrix elements) and we can say that we have preserved the self-consistency in the pairing channel, in the same way as in the p-h one.

The CSB and CIB forces are included in our HF-BCS iterative procedure. These forces are parametrized as in Ref. [20], where they have been cast in a form similar to that of the Skyrme interaction. They had been already employed, under the form of a Yukawa function, in Ref. [21] and they have been shown to reproduce well the correct mass number dependence of the Coulomb displacement energies, as well as a number of values of isospin mixing in the ground state. Finally, they turned out to be important to account for the IAR width in ^{208}Pb [16]. For all these reasons, we use these parametrizations in the present work. Because of their operatorial form, they do not add any contribution to the p-h force in RPA or QRPA. The electromagnetic spin-orbit is quite small: consequently, the associated energy shift has been added to the HF-BCS results using first-order perturbation theory.

III. RESULTS

The systematic trend of the IAR energies in the nuclei we have considered, $^{104-132}$Sn, is plotted in Fig. 2. The energies are obtained within QRPA, by including all the terms mentioned in the previous Section: only the proton-rich ^{104,106}Sn have been calculated using quasiparticle Tamm-Dancoff approximation (QTDA) because of QRPA instabilities. Our findings are compared with the experimental energies quoted in Ref. [22], where the results of the (^{3}He,t) reaction performed at an incident beam energy of 200 MeV are reported. It can be immediately realized that the agreement is fairly good. The difference between theory and experiment is typically ≈ 200 keV in the series of isotopes which have been measured, namely $^{112-124}$Sn (with the exception of the two extremes ^{112}Sn and ^{124}Sn where this difference is larger). It is remarkable that another microscopic, self-consistent model like RMF — which starts from a quite different description of the nuclear mean-field and its oscillations as already stressed in the Introduction — produces a similar numerical outcome [10]. The results for the IAR in the unstable nuclei do constitute a useful guideline for possible future experiments.

Concerning the results in the $(N+1,Z-1)$ channel, unfortunately few experimental measurements are available for a comparision with our model. The only exception is the

case of ^{120}Sn. In Fig. 3 we plot for this nucleus the response to the IVGMR operator,

$$\hat{O}_{\rm IVGMR} \equiv \sum_{i=1}^{A} r_i^2 t_+(i), \tag{6}$$

as a function of the energy with respect to the ground state of ^{120}In. The continuous curve has been obtained by averaging the QRPA discrete strength distribution with a 1 MeV width Lorentzian curve. We can compare our results with three experiments carried out by means of different nuclear reactions. By using (π^-, π^0) at 165 MeV [23], (^{13}C, ^{13}N) at 50 MeV/A [24] and (^{7}Li, ^{7}Be) at 350 MeV [25] it has been shown, more or less ambiguously, that a 0^+ state should lie, respectively, at 16.0±2.2 MeV, 14.7±1 MeV and 17.0±1.6 MeV. In our calculation most of the strength is found indeed in the energy region between 12 and 20 MeV. Our main peak seems compatible with the (^{7}Li, ^{7}Be) result.

Coming back to the case of the IAR, we analyze in more detail our results in order to clarify the most important features of our theoretical description. Firstly, in analogy with the conclusion drawn in Ref. [10], we may show that also in the present case the consistent treatment of pairing correlations is very important. In Fig. 4 we display three different results obtained for the IAR strength distribution in ^{114}Sn. Not only the residual proton-neutron pairing force plays a crucial role to concentrate the IAR in a single peak; it also affects the IAR energy in an important way, that is, it induces a downward shift of about 500 keV. In the whole isotopic series we have studied, the peak associated with the IAR exhausts typically a percentage between 95% and 98% of the $N - Z$ sum rule. Only in the isotope ^{108}Sn the IAR is found to be split in two peaks.

Having assessed the importance of the proton-neutron residual pairing, we have also tried to understand the role played by various other correlations present in our model. For this purpose, we display in Fig. 5 results for the IAR energy in ^{120}Sn obtained by making different approximations. The first number on the left side refers to a simple TDA calculation, without any pairing, without the spin-orbit term in the residual p-h force, and without the other terms which have been often neglected (electromagnetic spin-orbit, CSB and CIB). This would be the simplest possible calculation, analogous to that performed for many closed-shell nuclei in the previous literature. The inclusion of RPA ground state correlations do not affect very much the IAR, as it is expected for a nucleus which has neutron excess; the effect is larger if we move towards the neutron-deficient isotopes. Pairing correlations are more important. We have discussed above that they have to be included consistently (we

repeat that a calculation with pairing only in the ground state would lead to a too high, and fragmented, IAR): moreover, their inclusion shifts the IAR downwards by about 150 keV. At this stage, the QRPA result would differ from the experimental finding by about 500 keV. This would be approximately true for all the stable isotopes. The two-body spin-orbit have a non negligible effect (about 100 keV) in pushing the IAR energy towards the experimental value. Even more important, from a quantitative point of view, are in this case the CSB and CIB forces which are inserted in the HF-BCS calculation (the fact that they have opposite sign has been already remarked [8]). Finally, we have included for the sake of completeness the one-body electromagnetic spin-orbit. This term has also been calculated long time ago (see, e.g, p. 494 of Ref. [26]) and it is known to have, as a rule, an effect of only few tenths of keV on the Coulomb displacement energies. Because of its j-dependence, it may become significant in the case of pure transitions associated with large angular momentum, as it has been stressed in [27]. We should add that we have checked that the contributions stemming from the CSB, CIB and the electromagnetic spin-orbit are almost constant over the isotopic chain. In this sense, the numbers presented in Fig. 5 are considered as typical. As far as the two-body spin-orbit is concerned, in the middle of the chain the associated repulsive contribution is maximum; at the extremes of the chain it becomes smaller or even attractive (for instance, in ^{132}Sn we find an attractive contribution associated with the diagonal $h_{11/2}$ matrix element).

Since many Skyrme parametrizations are available in the market, we would like to mention that our results are not very sensitive to the choice of a specific set. In fact, we have seen that the IAR energy of ^{120}Sn varies by less than 100 keV if we calculate it either using the force SLy4, or SIII [28] or SGII [29]. We have also performed a calculation using the recently introduced SkO' interaction [30], in view of the possibility of testing it in the next future on the systematics of spin states. In this case, the variation of the energy, with respect to the result obtained by using SLy4, is somewhat larger [31]. Also the effect of varying the pairing strength V_0 has been considered, and we refer to Fig. 6 for the results obtained in the case of ^{116}Sn (i.e., the isotope in the middle of the 50–82 neutron shell). We can consider as satisfactory that variations of V_0 in the range $\approx$ 650–710 MeV fm^3, which lead to sizeable ($\approx$20%) variations of Δ, do not seriously affect the energy of the IAR. We can quite generally conclude that the choice of parameters, both of our p-h and p-p forces, do not seriously affect our main conclusions on IAR.

IV. CONCLUSIONS

Very few examples of microscopic, fully self-consistent charge-exchange QRPA calculations exist (in contrast with the non charge-exchange case). This has motivated the present work, in which we have developed the method and analyzed some specific issues: the relation between the isospin invariance and the self-consistency in the pairing channel, and the role of the usually neglected contributions in the mean-field. We have applied our scheme for the calculation of the IAR along the chain of the Sn isotopes. Only calculations based on RMF are available for this case. We find that our non-relativistic model can account quite well for the experimental results.

We plan to extend our calculations, and make further analysis of the charge-exchange states. This will be done for different multipolarities, both in the non spin-flip and spin-flip sectors. It is hoped that the comparison with experimental data, and with the outcome of other microscopic models, can be instrumental to fix rather general problems. In fact, as stressed in our Introduction, many uncertainties plague the isovector channel of the effective *NN* interaction, and consequently the symmetry part of the nuclear equation of state.

A possible improvement of our model consists in changing the description of the nuclear ground state, which may be calculated within full HFB instead of HF-BCS. This could allow a better description in the case, for instance, of weakly bound systems. Another open problem is the consideration of the role played by the proton-neutron pairing. Literature reflects the existence of many different thoughts about this interesting issue; a full microscopic QRPA calculation in the case in which the particles do not have a definite charge state may probably be at present too demanding. Finally, we should mention that the extension beyond mean-field of theories like ours remains to be done.

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[31] However, we must stress that this is the only calculation of our work which is characterized by some (arguably small) self-consistency breaking. In fact, the calculation of the ground state includes the spin-gradient terms (the so-called terms in J^2, $\vec{J}$ being the spin density) while the corresponding p-h residual force is omitted.

FIG. 1: The values of the pairing gaps Δ in the Sn isotopes. The open squares correspond to the empirical values, extracted by attributing to the isotope with N neutrons the value which results from the three-point formula centered in $N + 1$. The black squares correspond to the theoretical results: in this case, the values of the state-dependent HF-BCS pairing gaps Δ_i are averaged in an energy interval centered at the neutron Fermi energy and having a width of ± 5 MeV.

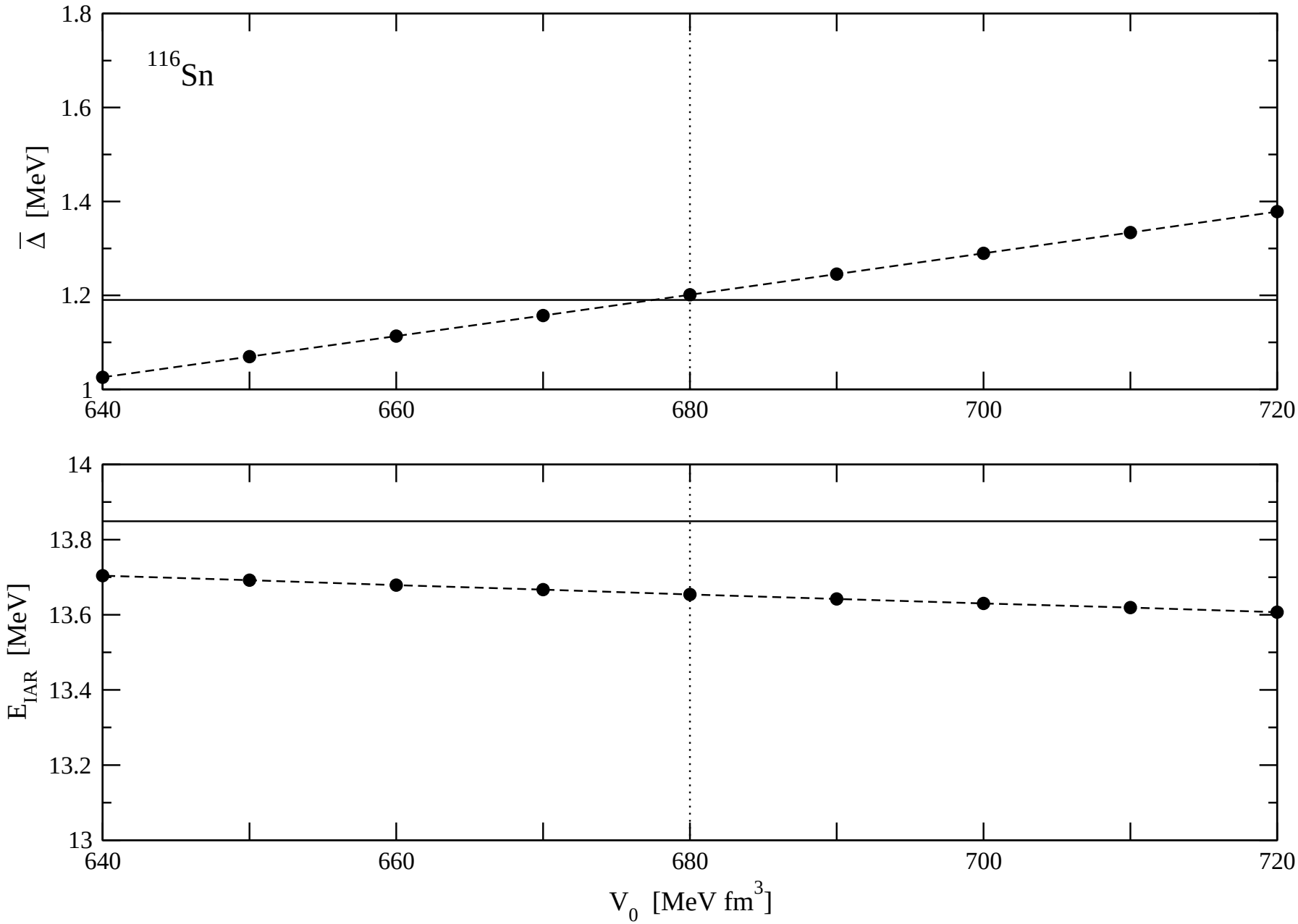
FIG. 2: Systematic trend of the IAR energies in the stable and unstable Sn isotopes. The theoretical results, displayed by means of black circles, are compared with the experimental data (open squares) whenever these are available. It must be noticed that the energies are referred to the daughter nuclei.

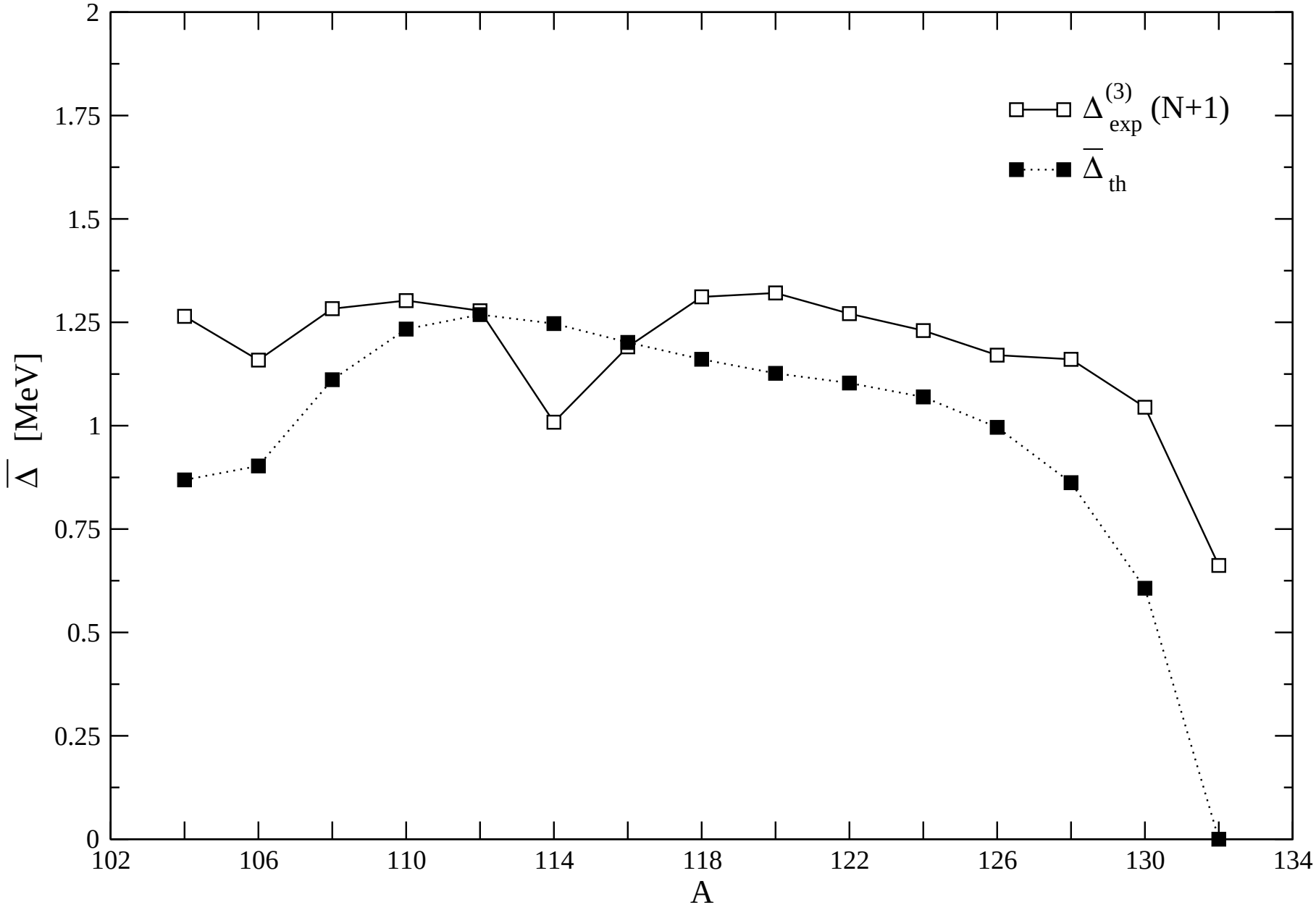
FIG. 4: Importance of the residual proton-neutron p-p interaction for the collectivity of the IAR. The left, central and right panels refer respectively to RPA, QRPA without that term in the residual force, and complete QRPA. The result is analogous to the one shown in Fig. 5 of Ref. [10].

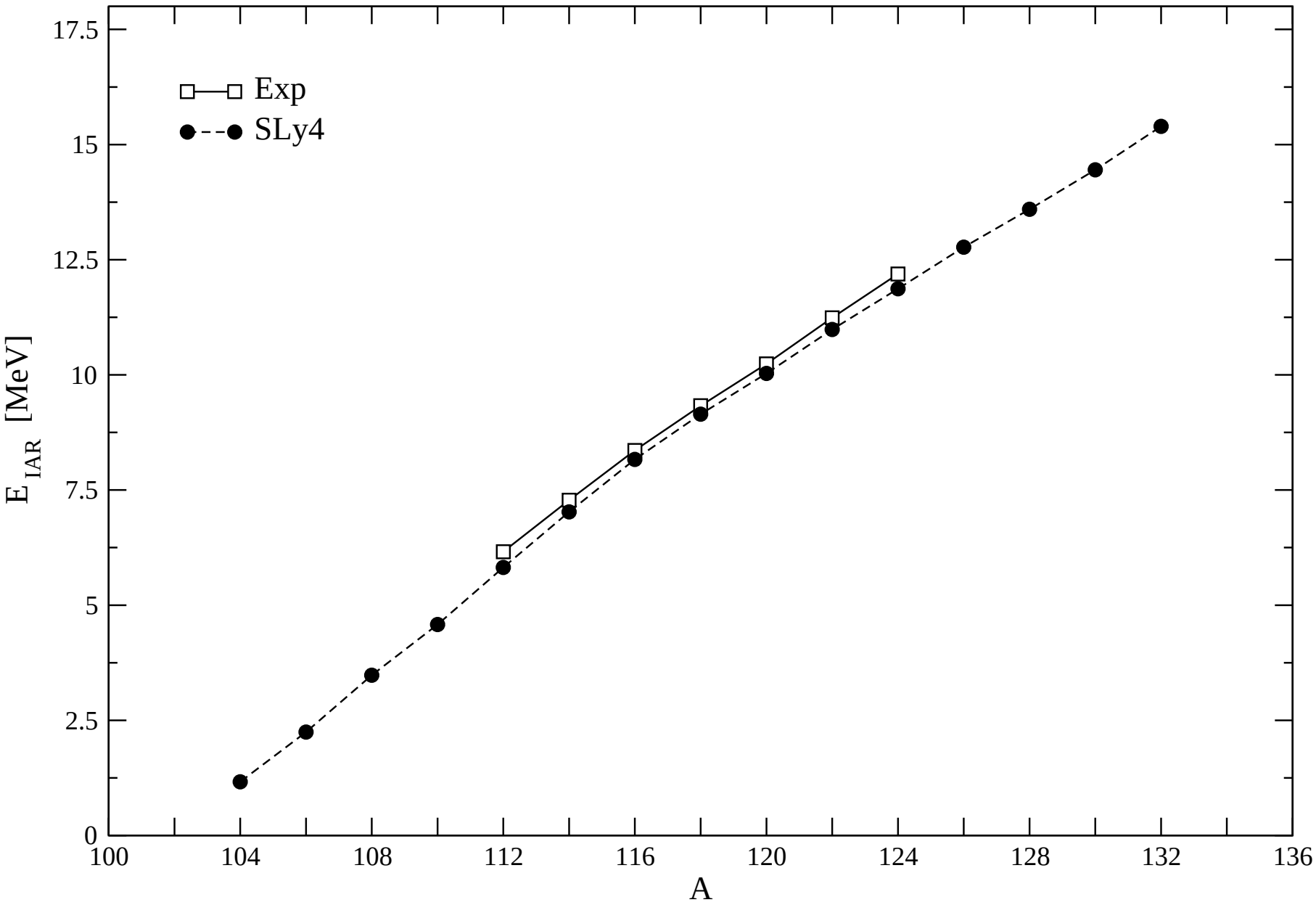
FIG. 5: Result for the IAR energy in ^{120}Sn obtained using different approximations. The values labelled by Δ represent the energy shifts of the IAR (in keV) at each step. See the text for a detailed discussion.

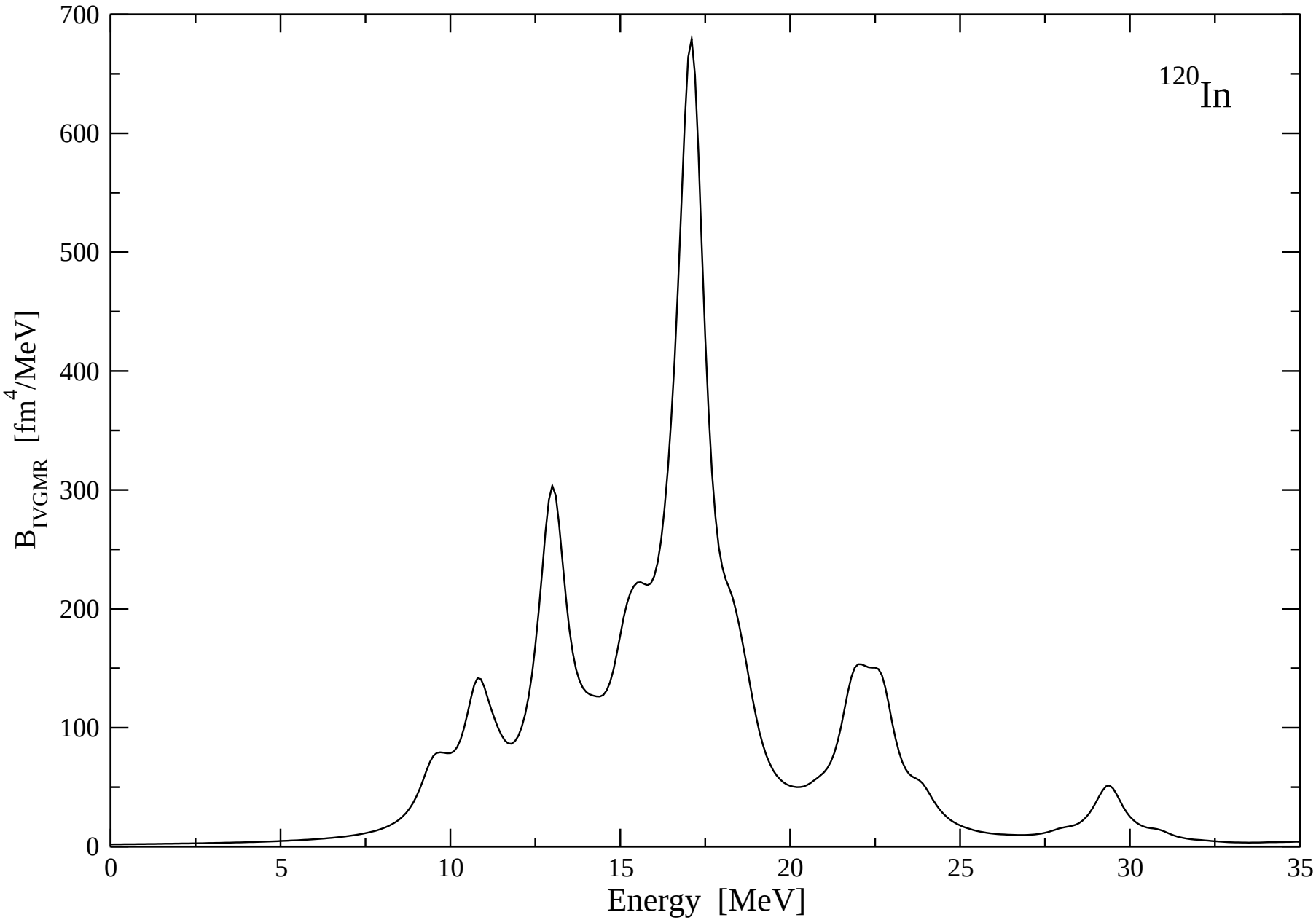
FIG. 6: Effect of the overall pairing strength V_0 which defines the effective force (3) on the pairing gap (upper panel) and the IAR energy (lower panel) in ^{116}Sn. The experimental values are marked by horizontal full lines, whereas the vertical dashed line indicates the adopted value of V_0.

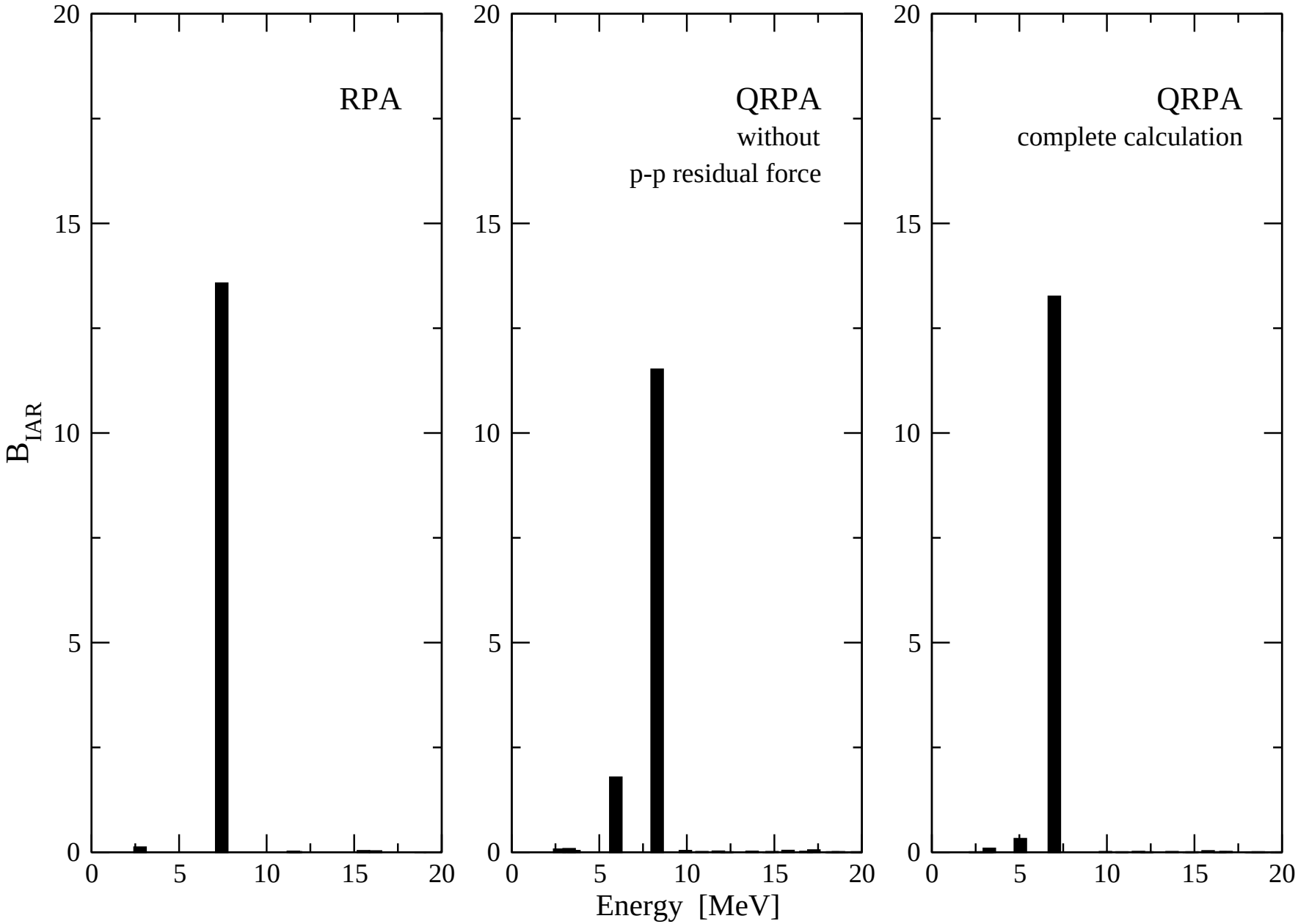
FIG. 3: Strength function associated with the IVGMR operator (6) in ^{120}In. The discrete QRPA peaks have been smoothed by using a Lorentzian averaging (the Lorentzian width is 1 MeV). See the text for a comparison with the available experimental results.

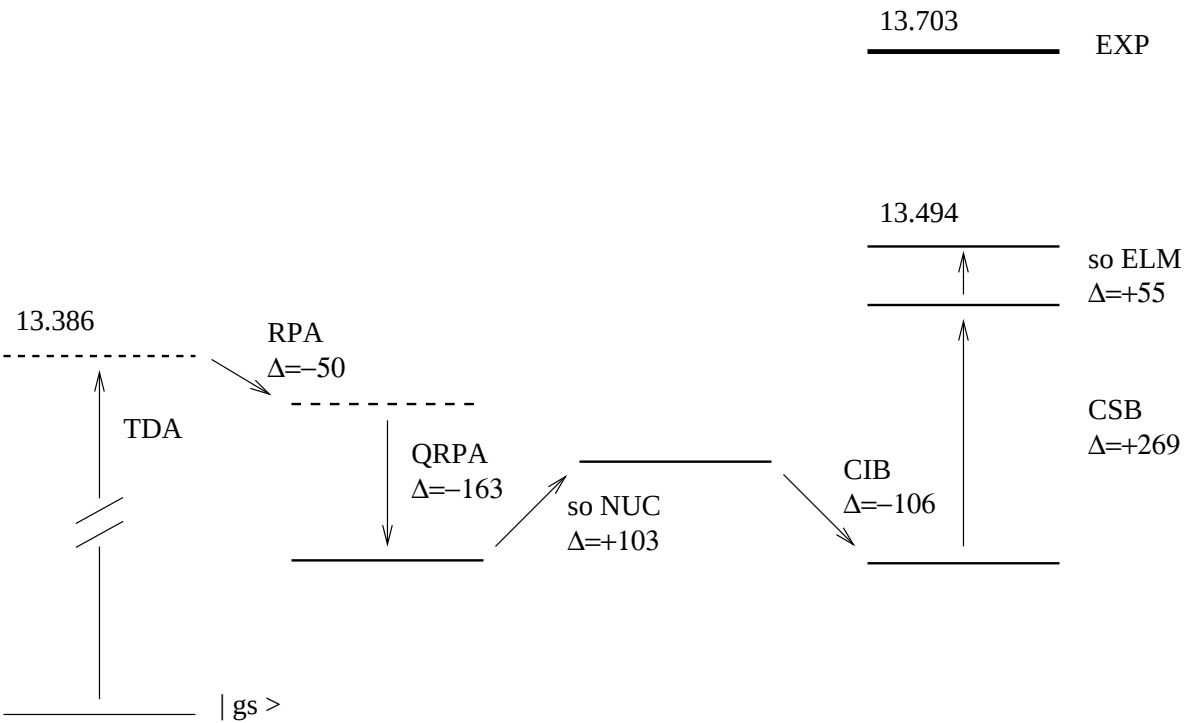

^{116}Sn
$\overline{\Delta}$ [MeV]
1.8
1.6
1.4
1.2
1
640
660
680
700
720
E_{IAR} [MeV]
14
13.8
13.6
13.4
13.2
13
V_0 [MeV fm^3]


2
1.75
1.5
1.25
1
0.75
0.5
0.25
0
102
106
110
114
118
122
126
130
134
A
$\overline{\Delta}$ [MeV]
$\Delta^{(3)}_{exp}$ (N+1)
$\overline{\Delta}_{th}$


Exp
SLy4
E_{IAR} [MeV]
A
0
2.5
5
7.5
10
12.5
15
17.5
100
104
108
112
116
120
124
128
132
136


^{120}In
B_{IVGMR} [fm^4/MeV]
Energy [MeV]
0
100
200
300
400
500
600
700
0
5
10
15
20
25
30
35


RPA
QRPA
without
p-p residual force
QRPA
complete calculation
B_IAR
Energy [MeV]
0
5
10
15
20


13.703
EXP
13.494
so ELM
Δ=+55
13.386
RPA
Δ=−50
TDA
QRPA
Δ=−163
so NUC
Δ=+103
CIB
Δ=−106
CSB
Δ=+269
| gs >